# Geometry, optimization and control in robot coordination

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Motion coordination is a remarkable phenomenon in biological systems such as schools of fish and serves as a remarkable tool for man-made groups of robotic vehicles and active sensors. Although each individual agent has no global knowledge about the group as a whole or about the surrounding environment, complex coordinated behaviors emerge from local interactions. From a scientific point of view, the study of motion coordination poses novel challenges in systems theory: a comprehensive understanding of this phenomenon requires the joint ambitious study of mobility, communication, computation, and sensing aspects.

This brief abstract and the presentation at the NE{S|T}COC symposium is a summary of some recent work on models and algorithms for coordinating the motion of multi-agent networks.

In the companion papers (Martínez, Bullo, Cortés, and Frazzoli, 2007), we propose a *tractable and meaningful model for a network of robotic agents* that move and communicate. Building on concepts from distributed computation, robotics and control theory, we define notions of robotic network, control and communication law, coordination task, and time and communication complexity. We illustrate our model and compute the proposed complexity measures in the prototypical example of a network of locally connected agents on a circle that agree upon a direction of motion and pursue their immediate neighbors. Additionally, we analyze a number of basic coordination algorithms: we provide upper and lower bounds on the time complexity of the move-toward average and circumcenter laws, both achieving rendezvous, and of the centroid law, achieving deployment over a region of interest. The results are derived via novel analysis methods, including a set of results on the convergence rates of linear dynamical systems defined by tridiagonal Toeplitz and circulant matrices.

In the article (Smith and Bullo, 2009), we study a target assignment problem consisting of an equal number of mobile robotic agents and distinct target locations dispersed in an environment. Each agent has a limited communication range, a maximum speed, and knowledge of every target's position. The problem is to devise a distributed algorithm that allows the agents to divide the target locations among themselves and, simultaneously, leads each agent to its unique target. We introduce distributed algorithms for solving the target assignment problem, and study their worst-case asymptotic performance; that is, the task completion time as the number of agents (and targets) increases, and the size of the environment scales to accommodate them. We introduce an intuitive class of algorithms for solving the target assignment problem, which we call *monotonic algorithms*, and give a lower bound on its performance. We design and analyze two algorithms within this class, called ETSP Assigment and Grid Assigment. In "sparse environments," where communication is infrequent, the ETSP Assigment algorithm is asymptotically optimal in the class of monotonic algorithms. We then show that in "dense environments," where communication is more prevalent, the Grid Assigment algorithm is asymptotically optimal in the class of monotonic algorithms. We discuss several extensions of our algorithms: The Grid Assigment algorithm also solves a sensor based version of the target assignment problem where agents initially have no information of target positions, but acquire them through a limited range sensor; and, both algorithms extend to problems with different numbers of targets and agents.

In the article (Pavone, Frazzoli, and Bullo, 2009), we present adaptive and distributed algorithms for motion coordination of a group of $m$ vehicles. The vehicles must service demands whose time of arrival, location and on-site service are stochastic; the objective is to minimize the expected system time of the demands. The general problem is known as the $m$-vehicle Dynamic

Traveling Repairman Problem ($m$-DTRP). The best previously known control algorithms rely on centralized task assignment and are not robust against changes in the environment. The contribution of this paper is threefold. First, we present new policies for the 1-DTRP that: (i) are provably optimal both in light- and heavy-load conditions, and (ii) are adaptive, in particular, they are robust against changes in load conditions. Second, we show that partitioning policies, whereby the environment is partitioned among the vehicles and each vehicle follows a certain set of rules in its own region, are optimal in heavy-load conditions. Finally, we design algorithms for the $m$-DTRP that (i) are adaptive and distributed, and (ii) are within a constant of the optimal (unbiased) performance in heavy-load conditions and stabilize the system in any load condition.

Finally, in the article (Frasca, Carli, and Bullo, 2009), we proposed novel algorithms for deployment, coverage and partitioning. Recently proposed algorithms achieve these tasks under a critical assumption: information is exchanged synchronously among all agents and long-range communication is possibly required. This work proposes novel deployment and partitioning algorithms that require only asynchronous pairwise (so-called gossip) communication. Which robot pair communicates at any given time may be selected deterministically or randomly. A key novel idea is the description of the coverage control problem as a control system on the space of partitions – in other words, we study the evolution of the regions assigned to each agent, rather than the evolution of the agents' positions. The novel gossip algorithms are shown to converge to multicenter Voronoi partitions through various results of independent interest: we establish the compactness of the space of partitions, the continuity of certain geometric maps (e.g., the Voronoi and the centroid maps), and two convergence theorems for switching dynamical systems on metric spaces.

In summary, it is worth mentioning that a unified treatment of these problems is presented in the recent subsequent textbook (Bullo, Cortés, and Martínez, 2009). The text is freely available at `http://coordinationbook.info`, where slides, software and bibliographical information are also available.

## References

Bullo, F., Cortés, J., and Martínez, S. (2009) *Distributed Control of Robotic Networks*, Applied Mathematics Series, Princeton University Press, ISBN 978-0-691-14195-4.

Frasca, P., Carli, R., and Bullo, F. (2009) Multiagent coverage algorithms with gossip communication: control systems on the space of partitions, in *American Control Conference*, St. Louis, MO, to appear.

Martínez, S., Bullo, F., Cortés, J., and Frazzoli, E. (2007) On synchronous robotic networks – Part I: Models, tasks and complexity notions. & Part II: Time complexity of rendezvous and deployment algorithms, *IEEE Transactions on Automatic Control*, **52**(12), 2199–2213 and 2214–2226.

Pavone, M., Frazzoli, E., and Bullo, F. (2009) Distributed and adaptive algorithms for vehicle routing in a stochastic and dynamic environment, *IEEE Transactions on Automatic Control*, submitted.

Smith, S. L. and Bullo, F. (2009) Monotonic target assignment for robotic networks, *IEEE Transactions on Automatic Control*, **54**(10), (Submitted June 2007) to appear.