Constructive Approaches to Submanifold Stabilization

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Abstract

We analyze control problems in which a quantity of interest must be brought towards a submanifold of the space in which it evolves. These problems arise in applications such as synchronization, pattern generation, or path following. In our approaches, we focus on constructive solutions to such problems. First, we attempt to bring the controlled system to the form of a gradient system with drift. Under the assumption that the scalar field defining the gradient vector field has strongly convex restrictions to the affinely translated normal spaces of the submanifold, we find that the gradient vector field can be scaled so as to ensure asymptotic stability of the submanifold. This condition further leads to an algebraic characterization of asymptotic stabilizability, expressed in terms of a Riemannian metric. The latter is of particular interest for systems evolving on Riemannian manifolds. As the aforementioned controls usually consume large amounts of energy, we next concentrate on optimal submanifold stabilization in the sense that we ask to bring a quantity towards a desired submanifold whilst, at the same time, maintaining control energy small. It turns out that optimal controls, in this context, are necessarily structured in such a way that they are state feedbacks which depend linearly on a function that maps to the normal spaces of the submanifold while the matrix describing this linear relationship contains the tangent spaces of the submanifold in its nullspace. Under additional assumptions, particular structured controls are also sufficient for optimality. Last, we solely require control energy and integral distance of a quantity to the submanifold to remain finite. To this end, we formulate a concept for input-output considerations of submanifold stabilization and therein exploit small gains, conicity, and passivity for controller design. Throughout, we illustrate our results by various examples.

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